Suggested Solutions of HW1

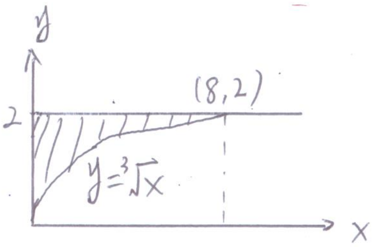
Ex. 15.1

20. $\iint_R \frac{y}{x^2y^2+1}\,dA = \int_0^1\int_0^1 \frac{y}{(xy)^2+1}\,dx\,dy = \int_0^1 (\tan^{-1}(xy))\Big|_0^1\,dy$

$= \int_0^1 \tan^{-1}y\,dy = \left(y\tan^{-1}y - \frac{1}{2}\ln|1+y^2|\right)\Big|_0^1 = \frac{\pi}{4}-\frac{1}{2}\ln 2$

Ex. 15.2

54. $\int_0^8\int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1}\,dy\,dx = \int_0^2\int_0^{y^3}\frac{1}{y^4+1}\,dx\,dy = \int_0^2 \frac{y^3}{y^4+1}\,dy.$




$= \frac{1}{4}(\ln(y^4+1))\Big|_0^2 = \frac{17}{4}$

58. $V = \int_{-2}^1\int_x^{2-x^2} x^2\,dy\,dx = \int_{-2}^1 (x^2y)\Big|_x^{2-x^2}\,dx = \int_{-2}^1 (2x^2-x^4-x^3)\,dx$

$= \left(\frac{2}{3}x^3-\frac{1}{5}x^5-\frac{1}{4}x^4\right)\Big|_{-2}^1 = \frac{63}{20}$

66. $V = 4\int_0^{\frac{\pi}{3}}\int_0^{\sec x}(1+y^2)\,dy\,dx = 4\int_0^{\frac{\pi}{3}}\left(y+\frac{y^3}{3}\right)\Big|_0^{\sec x}\,dx$

$= 4\int_0^{\frac{\pi}{3}}\left(\sec x+\frac{\sec^3 x}{3}\right)dx = \frac{2}{3}(7\ln|\sec x+\tan x|+\sec x\tan x)\Big|_0^{\frac{\pi}{3}}$

$= \frac{2}{3}[7\ln(2+\sqrt{3})+2\sqrt{3}]$

78. $\int_0^2(\tan^{-1}\pi x-\tan^{-1}x)\,dx = \int_0^2\int_x^{\pi x}\frac{1}{1+y^2}\,dy\,dx = \int_0^2\int_{\frac{y}{\pi}}^{y}\frac{1}{1+y^2}\,dx\,dy$

$+\int_2^{2\pi}\int_{\frac{y}{\pi}}^{2}\frac{1}{1+y^2}\,dx\,dy = \int_0^2\frac{(1-\frac{1}{\pi})y}{1+y^2}\,dy + \int_2^{2\pi}\frac{(2-\frac{y}{\pi})}{1+y^2}\,dy$

$= \left(\frac{\pi-1}{2\pi}\right)(\ln(1+y^2))\Big|_0^2 + \left(2\tan^{-1}y+\frac{1}{2\pi}\ln(1+y^2)\right)\Big|_2^{2\pi}$

$= \left(\frac{\pi-1}{2\pi}\right)\ln 5 + 2\tan^{-1}2\pi - \frac{1}{2\pi}\ln(1+4\pi^2) - 2\tan^{-1}2 + \frac{1}{2\pi}\ln 5$

$= 2\tan^{-1}2\pi - 2\tan^{-1}2 - \frac{1}{2\pi}\ln(1+4\pi^2) + \frac{\ln 5}{2}$

Ex 15.3

20. Average value over the square $= \int_0^1 \int_0^1 xy\,dy\,dx = \int_0^1 (\frac{xy^2}{2})\Big|_0^1 dx$

$= \int_0^1 \frac{x}{2} dx = \frac{1}{4}$

Average value over the quater circle $= \frac{1}{(\frac{\pi}{4})} \int_0^1 \int_0^{\sqrt{1-x^2}} xy\,dy\,dx$

$= \frac{4}{\pi} \int_0^1 [\frac{xy^2}{2}]_0^{\sqrt{1-x^2}} dx = \frac{2}{\pi} \int_0^1 (x-x^3)dx$

$= \frac{1}{2\pi}$.

The average value over the square is larger.

24. $\int_0^1 \int_{y^2}^{2y-y^2} 100(y+1)\,dx\,dy = \int_0^1 [100(y+1)x]_{y^2}^{2y-y^2} dy = \int_0^1 100(y+1)(2y-2y^2)dy$

$= 200 \int_0^1 (y-y^3)dy = 200[\frac{y^2}{2} - \frac{y^4}{4}]_0^1 = 50$.

Ex. 15.4

18. $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2} dy\,dx = 4 \int_0^{\frac{\pi}{2}} \int_0^1 \frac{2r}{(1+r^2)^2} dr\,d\theta = 4 \int_0^{\frac{\pi}{2}} [-\frac{1}{1+r^2}]_0^1 d\theta = \pi$.

30. $A = \int_0^{2\pi} \int_0^{\frac{4}{3}\theta} r\,dr\,d\theta = \frac{8}{9} \int_0^{2\pi} \theta^2 d\theta = \frac{64}{27}\pi^3$

34. Average height $= \frac{4}{\pi a^2} \int_0^{\frac{\pi}{2}} \int_0^a r^2\,dr\,d\theta = \frac{4}{3\pi a^2} \int_0^{\frac{\pi}{2}} a^3 d\theta = \frac{2}{3}a$.